

# C. U. SHAH UNIVERSITY

## Winter Examination-2019

Subject Name : Advanced Mathematics

Subject Code : 2TE02AMT2

Branch: Diploma (All)

Semester : 2

Date : 12/09/2019

Time : 02:30 To 05:30

Marks : 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

**Q-1**                      **Attempt the following questions:**                      **(14)**

- a)  $\lim_{x \rightarrow 0} \frac{\tan 3x}{2x} = \underline{\hspace{2cm}}$   
(A)  $3/2$  (B)  $2/3$  (C)  $1/3$  (D)  $1/2$
- b)  $\lim_{x \rightarrow \infty} x[\sqrt[3]{7} - 1] = \underline{\hspace{2cm}}$   
(A) 0 (B)  $e^7$  (C)  $\log_e 7$  (D) 1
- c)  $\lim_{x \rightarrow 0} \left(1 + \frac{5}{x}\right)^x = \underline{\hspace{2cm}}$   
(A)  $e^5$  (B) e (C)  $e^{1/5}$  (D) None of these
- d) If  $f(x) = \cos x$  then  $f(2\pi) = \underline{\hspace{2cm}}$   
(A) 1 (B) 0 (C) -1 (D) 2
- e)  $\frac{d(e^{10x})}{dx} = \underline{\hspace{2cm}}$   
(A)  $10e^x$  (B)  $e^x$  (C)  $\frac{e^{10x}}{10}$  (D)  $10e^{10x}$
- f)  $\frac{d(\cos^{-1} x)}{dx} = \underline{\hspace{2cm}}$   
(A)  $\frac{-1}{\sqrt{1-x^2}}$  (B)  $\frac{1}{\sqrt{1-x^2}}$  (C)  $\frac{1}{1+x^2}$  (D)  $\frac{-1}{1+x^2}$
- g)  $\frac{d(\cot x)}{dx} = \underline{\hspace{2cm}}$   
(A)  $-\operatorname{cosec}^2 x$  (B)  $\sec^2 x$  (C)  $-\sin x$  (D)  $\sin x$
- h)  $\frac{d(\sec^2 x - \tan^2 x)}{dx} = \underline{\hspace{2cm}}$



(A) 2 (B) 1 (C) 0 (D) None of these

i)  $\int \frac{1}{\sqrt{x^2 + 9}} dx = \underline{\hspace{2cm}}$

(A)  $\cot^{-1} \frac{x}{3} + c$  (B)  $\tan^{-1} \frac{x}{3} + c$  (C)  $\log \left| x + \sqrt{x^2 + 9} \right| + c$

(D) none of these

j)  $\int \frac{1}{x} dx = \underline{\hspace{2cm}}$

(A)  $\log x + c$  (B)  $e^x + c$  (C) 1 (D) 0

k)  $\int_0^1 \frac{1}{1+x^2} dx = \underline{\hspace{2cm}}$

(A)  $\pi$  (B)  $\frac{\pi}{4}$  (C)  $\frac{\pi}{2}$  (D) None of these

l) If  $a = i + j + k$  then  $\hat{a} = \underline{\hspace{2cm}}$ .

(A)  $\frac{1}{\sqrt{2}}(1, 1, 1)$  (B)  $\frac{1}{\sqrt{3}}(1, 1, 1)$  (C)  $\sqrt{3}$  (D) none of these

m) If  $x = (1, 1, 1)$  and  $y = (1, 0, 0)$  then  $x - y = \underline{\hspace{2cm}}$ .

(A)  $(0, 1, 0)$  (B)  $(0, 0, 1)$  (C)  $(1, 0, 0)$  (D) None of these

n) If  $\bar{x} = 2i - j + k$  and  $\bar{y} = i + 2j$  then  $\bar{x} \cdot \bar{y} = \underline{\hspace{2cm}}$ .

(A) 1 (B) 0 (C) -1 (D) -2

**Attempt any four questions from Q-2 to Q-8**

**Q-2**

**Attempt all questions**

**(14)**

a) If  $\bar{a} = 3i - 2j + k$ ,  $\bar{b} = 2i - 4j - 3k$  and  $\bar{c} = -i + 2j + 2k$

**(5)**

then find modulus of  $2\bar{a} - 3\bar{b} - 5\bar{c}$ .

b) If  $\bar{a} = i + j + k$  and  $\bar{b} = 2i - 2j + k$  then find unit vector

**(5)**

perpendicular to  $\bar{a}$  and  $\bar{b}$ .

c) Evaluate:  $\lim_{x \rightarrow 3} \frac{x^2 - x - 6}{x^3 - 3x^2 + x - 3}$

**(4)**

**Q-3**

**Attempt all questions**

**(14)**

a) Prove that  $\lim_{x \rightarrow a} \frac{\sqrt{2a-x} - \sqrt{x}}{a-x} = \frac{1}{\sqrt{a}}$ .

**(5)**

b) Evaluate:  $\lim_{x \rightarrow \frac{\pi}{4}} \frac{2 - \sec^2 x}{1 - \tan x}$

**(5)**

c) Find  $\frac{dy}{dx}$  if  $y = \frac{a + b \sin x}{a \sin x + b}$ .

**(4)**

**Q-4**

**Attempt all questions**

**(14)**

a)  $S = t^3 - 6t^2 + 9t + 6$  gives the distance travelled by a body in  $t$  seconds.

**(5)**

Find velocity and acceleration at  $t = 4$  seconds.



b) Find  $\frac{dy}{dx}$  if  $y = \log \sqrt{\frac{a+x}{a-x}}$ . (5)

c) If  $\bar{a} = 2i - j$ ,  $\bar{b} = i + 3j - 2k$  then obtain  $\left|(\bar{a} + \bar{b}) \times (\bar{a} - \bar{b})\right|$ . (4)

**Q-5**

**Attempt all questions**

(14)

a) Prove that angle between two vectors  $i + j - k$  and  $2i - 2j + k$  is (5)

$$\sin^{-1}\left(\frac{\sqrt{26}}{3\sqrt{3}}\right).$$

b) If  $y = 2e^{-3x} + 3e^{2x}$  then prove that  $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$ . (5)

c) Evaluate:  $\lim_{x \rightarrow \infty} \left[ \frac{x+1}{x+2} \right]^x$  (4)

**Q-6**

**Attempt all questions**

(14)

a) A particle moves from the point  $-i - j - k$  to the point  $k + j + i$  under the effect of two constant forces  $2i + j + k$  and  $i + 3j + k$ . Find the work done. (5)

b) Prove that  $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sec x}}{\sqrt{\sec x} + \sqrt{\cos ecx}} dx = \frac{\pi}{4}$ . (5)

c) Find derivative of  $y = x^3 - 2x$  using first principle. (4)

**Q-7**

**Attempt all questions**

(14)

a) Evaluate:  $\int \frac{e^x(1+x)}{\cos^2(xe^x)} dx$  (5)

b) Find the area of circle  $x^2 + y^2 = r^2$ . (5)

c) For what value of  $m$ , vectors  $mj + 2i + k$  and  $2i + 4j + 5k$  are perpendicular to each other? (4)

**Q-8**

**Attempt all questions**

(14)

a) Evaluate:  $\int x^n \log x dx$  (5)

b) Find  $\frac{dy}{dx}$  if  $y = (e^{3x} + 1)^{2x+5}$ . (5)

c) Evaluate:  $\int_0^{\pi/3} \frac{\sin x}{3+4\cos x} dx$  (4)

